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Attitude Stability Criteria for Dual Spin Spacecraft

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Attitude stability criteria are developed for "dual spin" spacecraft, which consist of two primary bodies capable of unlimited relative rotation about a common axis. Such vehicles, typified in existing hardware by the Orbiting Solar Observatory (OSO) satellites, have applicability to missions for which the simplicity, reliability, and longevity of spin stabilization combine with a requirement for unidirectional pointing of a component, such as an antenna or a solar panel. The utility of dual spin systems is substantially increased by the results of attitude stability analysis, which reveals the possibility of obtaining passive stable spin-axis attitude by spinning a vehicle about its axis of greatest or least inertia, providing only that an effective energy dissipation device is attached to a counter-rotating platform which has greatly reduced "spin." This result is contrary to common interpretation of the familiar "major axis spin" requirement for stability, according to which energy dissipation in a vehicle rotating about its axis of least inertia must produce instability. Stability criteria are developed first by Routhian analysis of a system with energy dissipation capability in only one of the two primary bodies, and subsequently in more general (but less rigorous) terms for a fully dissipative vehicle. Numerical integration and model studies provide corroboration.

Introduction

THE influence of energy dissipation on the stability of motion of freely spinning bodies has been examined extensively in recent years, due to its relevance to problems of space vehicle attitude stability. Damping in the vehicle determines whether the nominal spinning motion is asymptotically stable (so initial infinitesimal perturbations attenuate and the system returns asymptotically to simple spin) or unstable (so initial infinitesimal perturbations increase beyond an arbitrary preassigned limit, and the "wobble" of the vehicle grows).

Particular emphasis has been given to the special case of slightly flexible, dissipative systems in which there appear neither motors nor rotating elements. A stability criterion for such systems was first established in 1957 (by V. D. Landon and R. N. Bracewell, independently); these results were dramatically confirmed by the unanticipated instability of Explorer I in February of 1958. In brief, the requirement for stability of such systems is spin about the centroidal axis of maximum moment of inertia, here called the "major axis." Since first publication of this stability criterion[1–3] the stability of freely spinning vehicles has received widespread attention in the aerospace industry, as evidenced by published papers, company reports, and flight hardware development programs. Many presently functioning satellites derive their primary orientation stability through spin, and each of these has been designed on the basis of this criterion. Most of the research emphasis has been on the development of methods for the quantitative determination of the influence of energy dissipation on the motion of spinning bodies,[4–6] and on the application of these methods to artificially damped vehicles spinning about their major axis. There has been very little attention given to the exploration of conditions under which a flexible, dissipative vehicle may be stable in spin about its minor axis (centroidal axis of minimum inertia); in fact, it has generally been assumed that minor axis spin stability is impossible for flexible, dissipative systems. This assumption seems to be a consequence of uncritical acceptance of the major axis spin requirement as a rule of thumb; there is no analytical basis for the indiscriminant application of this criterion to vehicles containing motors or rotating parts. (In a recent attempt[7] at rigorous proof of the major axis spin requirement, this class of vehicles is explicitly excluded.)

That the major axis spin stability criterion does not always apply to dual spin vehicles has been discovered independently by V. D. Landon and A. J. Iorillo, and apparently also by others more recently. Dissemination of early results was thwarted by the unfortunate rejection in 1962 of Landon's first paper, in which the problem was treated approximately by evaluating at the force-torque level stabilizing tendencies in dampers. A later attempt, published with B. Stewart,[8] employed the "energy-sink" approximation previously used in application to spinning satellites;[1,2,4,6] this analysis[8] is limited to fully axisymmetric dual spin vehicles with a massless energy dissipation "sink" in one of the bodies.

Concurrent development of the dual spin stabilization concept by Iorillo resulted in a presentation[9] in 1965 that extended scope to axisymmetric vehicles with damping in both bodies. This extension is important because without it one cannot establish the feasibility of the dual spin stabilization concept for space vehicles of realistic complexity. In this paper other analytical approximations are introduced to support the results of an energy sink analysis.

The present paper is a brief report on the results of an intensive study with the objective of establishing more rigorously and more generally the criteria for attitude stability of dual spin systems.[10,11] All published studies had employed approximations that were subject to argument, and had been restricted to fully axisymmetric vehicles. It has proven possible to remove analytically each of these objections individually, but not simultaneously. Accordingly, this paper includes both an entirely rigorous Routhian analysis of a specialized class of vehicles and an energy sink approximation of a more general vehicle idealization. Substantiation of results by digital computer numerical integration and by observation of a demonstration model is also noted.

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Routh Stability Analysis

Consider the specific dual spin system shown in Fig. 1 to consist of an asymmetric body A with an attached axisymmetric rotor S and a mass-spring-dashpot damper aligned with the spin axis and removed from it a distance b. When the damper mass m is in its neutral position (spring undeformed), the mass center of the total system defines the point O fixed in body A, and m lies on the principal axis of the total undeformed system designated by the unit vector $\hat{\mathbf{a}}_1$.† The rotor axis is fixed along the principal axis of A identified by $\hat{\mathbf{a}}_3$. As the mass m moves a distance Z in the direction of $\hat{\mathbf{a}}_3$, the system mass center (designated C) moves a distance $\bar{Z}$ from O in direction $\hat{\mathbf{a}}_3$, where

$$\bar{Z} = (m/M)Z \equiv \mu Z \qquad (1)$$

with M the total system mass and μ the mass ratio.

Equations of motion of this system are derived here in rather more general terms than this particular system requires, in order to permit easier visualization of the more general case. From first principles the rotational equations of motion of a completely general flexible body are written as a vector-dyadic equation involving integrals, and for the special case given these integrals are evaluated and scalar equations of rotation are written These equations are supplemented by two scalar equations describing the internal motion of rotor and damper mass. The total set of equations is satisfied by spin in inertial space of both the symmetric and asymmetric bodies (at different rates) about their common axis. Linearized variational equations are written for this motion, and these equations are subjected to a Routh stability analysis.

As a starting point, the equation

$$\mathbf{M} = \dot{\mathbf{h}} \qquad (2)$$

is accepted, where dot over a vector indicates the time derivative in inertial space, $\mathbf{M}$ is the applied moment about the system mass center C, and $\mathbf{h}$ is the system angular momentum about C.

By definition, the angular momentum about C is the integral over the system mass

$$\mathbf{h} = \int \mathbf{R} \times \dot{\mathbf{R}} dm = \int \mathbf{R} \times (\boldsymbol{\omega} \times \mathbf{R}) dm + \int \mathbf{R} \times \mathbf{R}' dm \qquad (3)$$

with $\mathbf{R}$ the vector from C to differential mass element dm, $\boldsymbol{\omega}$ the angular velocity of A in inertial space, and $\mathbf{R}'$ the time derivative of $\mathbf{R}$ in reference frame A.

The evaluaton of these integrals is complicated by the fact that $\mathbf{R}$ is a generic position vector relative to the system mass center C, with respect to which all parts of the body are moving. It is convenient to work with vectors relative to the point O, fixed in A at the point occupied by C when the system is undeformed. If $\mathbf{R}_0$ is the vector from C to O, and $\mathbf{r}$ is the generic vector from O to a point of the system,

$$\mathbf{R} = \mathbf{R}_0 + \mathbf{r} \qquad (4)$$

and after manipulation $\mathbf{h}$ becomes

$$\mathbf{h} = \int \mathbf{r} \times (\boldsymbol{\omega} \times \mathbf{r}) dm + [\mathbf{R}_0' + \boldsymbol{\omega} \times \mathbf{R}_0] \times M\mathbf{R}_0 + \int \mathbf{r} \times \mathbf{r}' dm = \mathrm{II} \cdot \boldsymbol{\omega} + M\mathbf{R}_0' \times \mathbf{R}_0 + \int \mathbf{r} \times \mathbf{r}' dm \qquad (5)$$

where the inertia dyadic II of the total system about point O is defined by this substitution.

If the total system consisted of the rigid body A, the last two terms in Eq. (5) would be zero, and the inertia dyadic would have constant elements in the vector basis $\hat{\mathbf{a}}_1$, $\hat{\mathbf{a}}_2$, $\hat{\mathbf{a}}_3$. In the general case at hand, flexible appendages on body A make the elements of II in this basis variables, although body A and symmetric rotors with axes and mass centers fixed in A make constant contributions to these elements of the dyadic.

† Unit vectors are identified by a caret ($\wedge$).

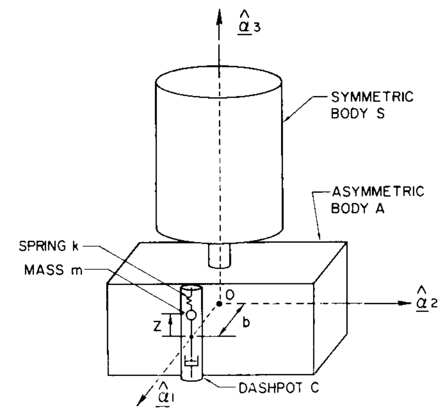

Fig. 1 Idealized dual spin system.

Construction of the equations of rotational motion requires only the inertial differentiation of $\mathbf{h}$; Eq. (2) becomes

$$\mathbf{M} = \mathrm{II}' \cdot \boldsymbol{\omega} + \mathrm{II} \cdot \dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathrm{II} \cdot \boldsymbol{\omega} + M[\mathbf{R}_0'' + 2\boldsymbol{\omega} \times \mathbf{R}_0' + \dot{\boldsymbol{\omega}} \times \mathbf{R}_0 + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{R}_0)] \times + \int \mathbf{r} \times \mathbf{r}'' dm + \boldsymbol{\omega} \times \int \mathbf{r} \times \mathbf{r}' dm \qquad (6)$$

Evaluation of the terms in the basic equation (6) is particularly simple for the system of Fig. 1; the contribution of a symmetric rotor to the two integrals becomes simply the inertial time derivative of the angular momentum of the rotor S relative to the body A, and the contributions of individual particles (such as the damper mass) are easily summed.

The rotor relative angular momentum can be written explicitly as

$$^{A}\mathbf{h}^{S} = I_3{}^{S}\Omega \mathbf{a}_3 \qquad (7)$$

where $I_3{}^S$ is the three-axis moment of inertia of the rotor and Ω is the angular speed of rotor S relative to body A, with the right-hand rule establishing the positive sense.

Then the rotor contributes to the basic equation of motion the term

$$^{A}\dot{\mathbf{h}}^{S} = I_3{}^{S}\dot{\Omega}\hat{\mathbf{a}}_3 + I_3{}^{S}\omega_2\Omega\hat{\mathbf{a}}_1 - I_3{}^{S}\omega_1\Omega\hat{\mathbf{a}}_2 \qquad (8)$$

The inertia dyadic II for the total system about point O may be written in the vector basis of body A as

$$\mathrm{II} = I_{\alpha\beta}\hat{\mathbf{a}}_\alpha\hat{\mathbf{a}}_\beta \qquad (9)$$

where repetition of Greek indices indicates summation from one to three. The moments and products of inertia for the total system, as indicated by doubly subscripted letters I, are calculated below; letters I with single subscripts are the three principal axis moments of inertia of the undeformed system about point O, the mass center of the system when undeformed

$$I_{11} = I_1 + mZ^2 \qquad I_{22} = I_2 + mZ^2 \qquad I_{33} = I_3$$
$$I_{12} = I_{21} = I_{23} = I_{32} = 0 \qquad I_{13} = I_{31} = -mbZ \qquad (10)$$

The equations of motion (6) involve also the term $\mathbf{R}_0$, which from (1) is

$$\mathbf{R}_0 = -\Omega Z \mathbf{a}_3 \qquad (11)$$

Substitution into the vector equation of motion (6) of Equations (8–11) and their derivatives provides the three scalar equations

$$I_1\dot{\omega}_1 - \omega_2\omega_3(I_2 - I_3) + I_3{}^{S}\Omega\omega_2 + m(1 - \mu)\dot{\omega}_1 Z^2 - m(1 - \mu)\omega_2\omega_3 Z^2 + 2m(1 - \mu)\omega_1\dot{Z}Z - mb\dot{\omega}_3 Z - mb\omega_1\omega_2 Z = 0 \qquad (12)$$

$$I_2\dot{\omega}_2 - \omega_3\omega_1(I_3 - I_1) - I_3{}^S\Omega\omega_1 + m(1-\mu)\dot{\omega}_2Z^2 + m(1-\mu)\omega_1\omega_3Z^2 + 2m(1-\mu)\omega_2Z\ddot{Z} - mb\ddot{Z} + mb\omega_1{}^2Z - mb\dot{\omega}_3{}^2Z = 0 \quad (13)$$

and

$$mb\omega_2\omega_3Z + I_3\dot{\omega}_3 + I_3{}^S\dot{\Omega} - \omega_1\omega_2(I_1 - I_2) - 2mb\omega_1\dot{Z} - mb\dot{\omega}_1Z = 0 \quad (14)$$

These three basic equations in the five unknowns ω_1, ω_2, ω_3, Ω, and Z must be supplemented by some internal specification of the behavior of the rotor and the damper mass to obtain a complete set of five scalar equations.

The rotor, by virtue of its symmetry, experiences a total angular rate in inertial space $\omega_3{}^S$ given by

$$I_3{}^S\dot{\omega}_3{}^S = T \quad (15)$$

with T the magnitude of the moment applied about the rotor axis or, since

$$\omega_3{}^S = \omega_3 + \Omega \quad (16)$$

the supplementary equation for the rotor is

$$I_3{}^S(\dot{\omega}_3 + \dot{\Omega}) = T \quad (17)$$

In this application, the only $\hat{\mathbf{a}}_3$-axis moment applied to the rotor is the combination of bearing friction and motor torque. In a satellite application of this system, the motor torque would normally be determined by a closed-loop control system designed to maintain a specified rate ω_3. If ω_3 is held at the orbital rate and $\hat{\mathbf{a}}_3$ is normal to the orbital plane, the asymmetric body maintains an earth-pointing attitude.

The damper mass motion in the $\hat{\mathbf{a}}_3$ direction is given by the $\hat{\mathbf{a}}_3$ component of the equation

$$\mathbf{F} = m\mathbf{a} \quad (18)$$

where $\mathbf{a}$ is the inertial acceleration of the damper mass point. The $\hat{\mathbf{a}}_3$ component of the force applied to the damper mass is simply that of the spring and the dashpot. Consequently Eq. (18) provides

$$m(1-\mu)\ddot{Z} + c\dot{Z} + kZ - m(1-\mu)(\omega_1{}^2 + \omega_2{}^2)Z + mb\omega_1\omega_3 - mb\dot{\omega}_2 = 0 \quad (19)$$

Equations (12–14, 17, and 19) constitute a complete set. They are satisfied by the solution

$$\begin{aligned} &\omega_3 = \omega_A && \text{a constant} \\ &\Omega = \Omega_S && \text{a constant (so } \omega_3{}^S = \omega_A + \Omega_S \equiv \omega_S) \\ &\omega_1 = \omega_2 = Z = 0 \text{ identically} && \end{aligned} \quad (20)$$

providing that motor torque balance bearing friction so $T = 0$.

To establish necessary and sufficient conditions for the asymptotic stability of this solution, one can construct the variational (perturbed) equations about solution (20), linearize these equations in the variational coordinates, and subject the linearized variational equations to a Routh stability analysis.

The variational coordinates are characterized by the symbols ω_1, ω_2, $\omega_A{}^*$, Z, and Ω^* (or $\omega_S{}^*$), so terms above the first degree in all these terms and their derivatives are dropped. The resulting linearized variational equations are written below:

$$I_1\dot{\omega}_1 + [(I_3 - I_2)\omega_A + I_3{}^S\Omega_S]\omega_2 = 0 \quad (21)$$

$$I_2\dot{\omega}_2 - [(I_3 - I_1)\omega_A + I_3{}^S\Omega_S]\omega_1 - mb\ddot{Z} - mb\omega_A{}^2Z = 0 \quad (22)$$

$$I_3\dot{\omega}_A{}^* + I_3{}^S\dot{\Omega}^* = 0 \quad (23)$$

$$I_3{}^S(\dot{\omega}_A{}^* + \dot{\Omega}^*) = T \quad (24)$$

$$m(1-\mu)\ddot{Z} + c\dot{Z} + kZ + mb\omega_A\omega_1 - mb\dot{\omega}_2 = 0 \quad (25)$$

Define the symbols

$$\begin{aligned} \lambda_1 &= [(I_3 - I_2)\omega_A + I_3{}^S\Omega_S]/I_1 = [I_3{}^A\omega_A + I_3{}^S\omega_S - I_2\omega_A]/I_1 \\ \lambda_2 &= [(I_3 - I_1)\omega_A + I_3{}^S\Omega_S]/I_2 = [I_3{}^A\omega_A + I_3{}^S\omega_S - I_1\omega_A]/I_2 \end{aligned} \quad (26)$$

Note that the angular momentum magnitude of the nominal motion in Eq. (20) is

$$h_0 = I_3{}^A\omega_A + I_3{}^S\omega_S \quad (27)$$

so that λ_1 and λ_2 may be written

$$\lambda_1 = (h_0 - I_2\omega_A)/I_1 \qquad \lambda_2 = (h_0 - I_1\omega_A)/I_2 \quad (28)$$

If the natural frequency of the damper in a fixed base is $p = (k/m)^{1/2}$, and the normalized constant of the dashpot is $\beta = c/m$, and further normalization is accomplished by the dimensionless parameter definitions $(Z/b) \equiv z$, and $(md^2/I_2) \equiv \delta$, the nontrivial linearized variational equations adopt the form

$$\begin{aligned} &\dot{\omega}_1 + \lambda_1\omega_2 = 0 \\ &\dot{\omega}_2 - \lambda_2\omega_1 - \delta\ddot{z} - \delta\omega_A{}^2z = 0 \\ &(1-\mu)\ddot{z} + \beta\dot{z} + p^2z + \omega_A\omega_1 - \dot{\omega}_2 = 0 \end{aligned} \quad (29)$$

In ignoring Eqs. (23) and (24), one assumes that the sum of bearing friction and motor torque is sufficiently small to justify the linearization in these equations. Strictly speaking, this sum must be arbitrarily small, although not precisely zero.

Either by Laplace transformation or by assuming an exponential solution, one can obtain from Eq. (29) the characteristic equation

$$s^4(1 - \mu - \delta) + s^3\beta + s^2[p^2 - \delta\omega_A{}^2 + \lambda_1\lambda_2(1-\mu) - \lambda_1\delta\omega_A] + s\beta\lambda_1\lambda_2 + \lambda_1\lambda_2p^2 - \lambda_1\delta\omega_A{}^3 = 0 \quad (30)$$

Routh has established consistency of algebraic sign of the coefficients of the foregoing s^n as a necessary condition for asymptotic stability.[12] Since for typical systems, $\delta \ll 1$, and $\mu \ll 1$, the only nontrivial consequence of this restriction is

$$\lambda_1\lambda_2 > 0 \quad (31)$$

To establish necessary and sufficient conditions for asymptotic stability from a characteristic equation of the form of Eq. (30) one can construct the Routhian array[12] and invoke the requirement that all entries in the first column be of the same sign (positive here).

The emerging necessary and sufficient conditions for asymptotic stability become, upon expanding the array,

$$\text{a) } 1 - \mu - \delta > 0 \quad (32)$$

$$\text{b) } \beta > 0 \quad (33)$$

$$\text{c) } \beta[p^2 + \lambda_1\lambda_2(1-\mu) - \delta\omega_A(\omega_A + \lambda_1)] - \beta\lambda_1\lambda_2(1 - \mu - \delta) > 0$$

or

$$p^2 - \delta(\omega_A{}^2 + \omega_A\lambda_1 - \lambda_1\lambda_2) > 0 \quad (34)$$

$$\text{d) } \lambda_1\lambda_2[p^2 + \lambda_1\lambda_2(1-\mu) - \delta\omega_A(\omega_A + \lambda_1)] - \lambda_1{}^2\lambda_2{}^2(1 - \mu - \delta) - \lambda_1\lambda_2p^2 + \lambda_1\delta\omega_A{}^3 > 0$$

or

$$\delta\lambda_1[\omega_A{}^3 + \lambda_1\lambda_2{}^2 - \lambda_1\lambda_2\omega_A - \lambda_2\omega_A{}^2] > 0$$

and since $\delta > 0$ by definition, this is

$$\lambda_1(\omega_A - \lambda_2)(\omega_A{}^2 - \lambda_1\lambda_2) > 0 \quad (35)$$

$$\text{e) } \lambda_1[\lambda_2 p^2 - \delta\omega_A{}^3] > 0 \quad (36)$$

The inequalities (32–36) are the objective of this section. They are stability conditions for the dual spin motion of the system shown in Fig. 1, without approximation or restriction. For practical systems, several of these inequalities are trivial. This will be illustrated by considering the limiting case of small damper mass. It is always assumed that $p^2 > 0$, $\beta > 0$, as must be the case for a real dashpot and real spring.

As $m \rightarrow 0$ (so $\mu \rightarrow 0$, $\delta \rightarrow 0$), the nontrivial conditions for stability of the system with small damper mass become

$$\text{d) } \lambda_1(\omega_A - \lambda_2)(\omega_A{}^2 - \lambda_1\lambda_2) > 0 \quad (37)$$

$$\text{e) } \lambda_1\lambda_2 > 0 \quad (38)$$

In order to interpret these inequalities, the definitions of λ_1 and λ_2 may be substituted from Eq. (28) and manipulated to provide for inequality (37)

$$\lambda_1 h_0[\omega_A(I_1 + I_2) - h_0]^2/I_1 I_2{}^2 > 0$$

or equivalently

$$\lambda_1 h_0 > 0 \quad (39)$$

At this point a sign convention is established which will be followed throughout; namely

$$h_0 \equiv I_3{}^A\omega_A + I_3{}^S\omega_S > 0 \quad (40)$$

With this convention, any of the nominal spin rates ω_A, ω_S, Ω_S may be negative or zero. The convention is equivalent to selecting the nominal direction of the unit vector $\hat{\mathbf{a}}_3$ as the direction of the angular momentum vector. With $h_0 > 0$, the stability conditions (38) and (39) in final form become

$$\lambda_1 > 0 \qquad \lambda_2 > 0 \quad (41)$$

For the special case of the fully axisymmetric dual spin system, inequality (41) becomes simply

$$\lambda \equiv (h_0 - I\omega_A)/I > 0 \qquad \text{or} \qquad h_0 > I\omega_A \quad (42)$$

where I is the transverse inertia of the system.

Another special case of interest is the "despun platform," for which $\omega_A = 0$. Then all the stability conditions are trivially satisfied for this system, which has damping on only the asymmetric (despun) part of the system. Note that this result is not what would be obtained by indiscriminant application of the major axis spin rule for stability; an axially elongated spinning body is here rendered asymptotically stable by the addition of a damper to a despun platform of arbitrary size and shape.

If, on the other hand, the asymmetric body containing the damper is spinning in inertial space, and the symmetric (undamped) body is at rest, the stability conditions become

$$\lambda_1 = [h_0 - I_2\omega_A]/I_1 = (I_3{}^A - I_2)\omega_A/I_1 > 0$$
$$\lambda_2 = (I_3{}^A - I_1)\omega_A/I_2 > 0$$

or equivalently

$$I_3{}^A - I_2 > 0 \qquad I_3{}^A - I_1 > 0 \quad (43)$$

which is the familiar major axis spin requirement, except that I_1 and I_2 include also the contribution of the rotor, if any is present.

The direct objective of this section has been the determination of stability conditions for the system of Fig. 1. The methods developed here for this purpose have been somewhat more comprehensive than the problem requires. This effort has been justified on the grounds that it facilitates visualization of more complex systems for which the asymmetric body A has more general flexibility and dissipation capability than this simple system permits. It is submitted that from this detailed example one can develop a rather broad understanding of the dynamics of dampers on spinning bodies. This thesis is developed in detail in the reports[10,11] previously mentioned. It may be noted here that extension is completely straightforward for a vehicle consisting of a rigid asymmetric body A with several attached particle dampers and several rigid rotors with axes of relative rotation fixed in A (although the rotor mass centers need not be fixed on a principal axis of A).

Energy-Sink Analysis

The results of the preceding section establish unequivocally the possibility in theory of obtaining asymptotically stable spin for a body rotating about its axis of least inertia. The applicability of those results to a physical system, which is flexible and dissipative in all its parts, is called into question however by the preceding conclusion that for an elongated spinning body (e.g., a spinning missile) with a despun platform, the presence of a damper on the platform alone makes the motion asymptotically stable, whereas alternative damper location in the spinning body itself results in instability. To determine the feasibility of the dual spin stabilization concept, one must confront the case in which damping is present throughout the system.

If equations of motion are constructed for a specific system with dampers on both bodies, it develops that the linearized variational equations have coefficients periodic in time. The Routh procedure is restricted to constant coefficient equations, and is thus inapplicable here. The only available alternative is a Floquet analysis,[13] and implementation of this analytical procedure generally requires numerical integration and cannot yield the desired algebraic stability criteria. Practical recourse is taken here to the informal but highly efficacious energy-sink method.

Initial calculations for this analysis are strictly applicable only to the simplest idealization; the system is a rigid asymmetric body with a rigid axisymmetric rotor attached. The equations of motion for such a system can be obtained by specializing Eq. (29) by setting $Z \equiv 0$:

$$\dot{\omega}_1 + \lambda_1\omega_2 = 0 \qquad \dot{\omega}_2 - \lambda_2\omega_1 = 0 \quad (44)$$

As in the previous section, equations involving the changes in rotation rates of the two bodies about their common axis are ignored, under the assumption of small motor and bearing friction torques. For convenience in manipulation without jeopardization of generality, assume the initial conditions

$$\omega_2(0) = \omega_0 \qquad \omega_1(0) = 0 \quad (45)$$

so that the solutions to (44) become

$$\begin{aligned}\omega_1 &= -\omega_0(\lambda_1/\lambda_2)^{1/2}\sin[(\lambda_1\lambda_2)^{1/2}t]\\ \omega_2 &= \omega_0\cos[(\lambda_1\lambda_2)^{1/2}t]\end{aligned} \quad (46)$$

Note the resulting necessary condition for stability:

$$\lambda_1\lambda_2 > 0 \quad (47)$$

The kinetic energy of rotation for this idealized dual spin system is

$$T = \tfrac{1}{2}I_1\omega_1{}^2 + \tfrac{1}{2}I_2\omega_2{}^2 + \tfrac{1}{2}I_3{}^A(\omega_3{}^A)^2 + \tfrac{1}{2}I_3{}^S(\omega_3{}^S)^2 \quad (48)$$

and the square of the system angular momentum about its mass center is

$$\mathbf{h}\cdot\mathbf{h} = I_1{}^2\omega_1{}^2 + I_2{}^2\omega_2{}^2 + (I_3{}^A\omega_3{}^A + I_3{}^S\omega_3{}^S)^2 \quad (49)$$

Thus far everything is precisely in accordance with linearization theory. Now, however, an energy dissipation mechanism, previously ignored, is permitted to function, so that approximately

$$\dot{T} = I_1\omega_1\dot{\omega}_1 + I_2\omega_2\dot{\omega}_2 + I_3{}^A\omega_3{}^A\dot{\omega}_3{}^A + I_3{}^S\omega_3{}^S\dot{\omega}_3{}^S < 0 \quad (50)$$

although still the angular momentum is constant, so

$$\tfrac{1}{2}(d/dt)(\mathbf{h}\cdot\mathbf{h}) = I_1{}^2\omega_1\dot{\omega}_1 + I_2{}^2\omega_2\dot{\omega}_2 + (I_3{}^A\omega_3{}^A + I_3{}^S\omega_3{}^S)(I_3{}^A\dot{\omega}_3{}^A + I_3{}^S\dot{\omega}_3{}^S) = 0 \quad (51)$$

Now of course the solutions (46) can no longer be precisely correct. If however, the effects of the energy dissipation mechanism are felt slowly, one may in the spirit of the variation of parameters approach assume solutions of the form of Eq. (46) but with $\omega_0 = \omega_0(t)$, a slowly varying function of time. Letting $\Lambda \equiv (\lambda_1/\lambda_2)^{1/2}$ and abbreviating $c \equiv \cos$, $s \equiv \sin$, one finds the derivatives

$$\begin{aligned}\dot{\omega}_1 &= -\dot{\omega}_0\Lambda s\Lambda\lambda_2 t - \omega_0\lambda_1 c\Lambda\lambda_2 t\\ \dot{\omega}_2 &= \dot{\omega}_0 c\Lambda\lambda_2 t - \omega_0\lambda_2\Lambda s\Lambda\lambda_2 t\end{aligned} \quad (52)$$

so that Eqs. (50) and (51) become, respectively,

$$\dot{T} = I_1\{\omega_0{}^2\lambda_1\Lambda s\Lambda\lambda_2 t c\Lambda\lambda_2 t + \omega_0\dot{\omega}_0\Lambda^2 s^2\Lambda\lambda_2 t\} + I_2\{-\omega_0{}^2\lambda_2\Lambda s\Lambda\lambda_2 t c\Lambda\lambda_2 t + \omega_0\dot{\omega}_0 c^2\Lambda\lambda_2 t\} + I_3{}^A\omega_3{}^A\dot{\omega}_3{}^A + I_S{}^S\omega_3{}^S\dot{\omega}_3{}^S \quad (53)$$

and

$$0 = I_1{}^2\{\omega_0{}^2\lambda_1\Lambda s\Lambda\lambda_2 t c\Lambda\lambda_2 t + \omega_0\dot{\omega}_0\Lambda^2 s^2\Lambda\lambda_2 t\} + I_2{}^2\{-\omega_0{}^2\lambda_2\Lambda s\Lambda\lambda_2 t c\Lambda\lambda_2 t + \omega_0\dot{\omega}_0 c^2\Lambda\lambda_2 t\} + (I_3{}^A\omega_3{}^A + I_3{}^S\omega_3{}^S)(I_3{}^A\dot{\omega}_3{}^A + I_3{}^S\dot{\omega}_3{}^S) \quad (54)$$

Averaging over the period $\tau = 2\pi/2\lambda_2\Lambda$ filters out all but the secular terms in Eqs. (53) and (54), assuming sufficiently slow time variations in ω_0. The result is the approximation

$$\bar{\dot{T}} = \tfrac{1}{2}I_1\omega_0\dot{\omega}_0(\lambda_1/\lambda_2) + \tfrac{1}{2}I_2\omega_0\dot{\omega}_0 + I_3{}^A\omega_3{}^A\dot{\omega}_3{}^A + I_3{}^S\omega_3{}^S\dot{\omega}_3{}^S \quad (55)$$

$$0 = \tfrac{1}{2}I_1{}^2\omega_0\dot{\omega}_0(\lambda_1/\lambda_2) + \tfrac{1}{2}I_2{}^2\omega_0\dot{\omega}_0 + (I_3{}^A\omega_3{}^A + I_3{}^S\omega_3{}^S)(I_3{}^A\dot{\omega}_3{}^A + I_3{}^S\dot{\omega}_3{}^S) \quad (56)$$

Solving for $\omega_0\dot{\omega}_0$ from Eq. (56) and substituting into $\bar{\dot{T}}$ yields the following:

$$\omega_0\dot{\omega}_0 = -2\lambda_2(I_3{}^A\omega_3{}^A + I_3{}^S\omega_3{}^S)(I_3{}^A\dot{\omega}_3{}^A + I_3{}^S\dot{\omega}_3{}^S)/(I_1{}^2\lambda_1 + I_2{}^2\lambda_2) \quad (57)$$

and

$$\bar{\dot{T}} = -I_3{}^A\dot{\omega}_3{}^A[-\omega_3{}^A + (I_3{}^A\omega_3{}^A + I_3{}^S\omega_3{}^S) \times (I_1\lambda_1 + I_2\lambda_2)/(I_1{}^2\lambda_1 + I_2{}^2\lambda_2)] - I_3{}^S\dot{\omega}_3{}^S[-\omega_3{}^S + (I_3{}^A\omega_3{}^A + I_3{}^S\omega_3{}^S) \times (I_1\lambda_1 + I_2\lambda_2)/(I_1{}^2\lambda_1 + I_2{}^2\lambda_2)] \quad (58)$$

The linearization previously employed permits the substitution of the nominal rates ω_A and ω_S for $\omega_3{}^A$ and $\omega_3{}^S$, respectively. Then, recalling the definition of h_0 and introducing the new definition

$$\lambda_0 = h_0(I_1\lambda_1 + I_2\lambda_2)/(I_1{}^2\lambda_1 + I_2{}^2\lambda_2) \quad (59)$$

one can write $\bar{\dot{T}}$ as

$$\bar{\dot{T}} = -I_3{}^A\dot{\omega}_3{}^A(\lambda_0 - \omega_A) - I_3{}^S\dot{\omega}_3{}^S(\lambda_0 - \omega_S) \quad (60)$$

With the further definitions

$$\lambda_A = \lambda_0 - \omega_A \qquad \lambda_S = \lambda_0 - \omega_S \quad (61)$$

$\bar{\dot{T}}$ takes the convenient form

$$\bar{\dot{T}} = -I_3{}^A\dot{\omega}_3{}^A\lambda_A - I_3{}^S\dot{\omega}_3{}^S\lambda_S \quad (62)$$

The two distinct parts of $\bar{\dot{T}}$ in (62) may be labeled

$$P_A = -I_3{}^A\dot{\omega}_3{}^A\lambda_A \qquad P_S = -I_3{}^S\dot{\omega}_3{}^S\lambda_S \quad (63)$$

and identified on physical grounds as the first approximation average energy dissipation rates (units of power) in bodies A and S, respectively. The validity of this identification is established by rewriting Eq. (63) as

$$I_3{}^A\dot{\omega}_3{}^A = -P_A/\lambda_A \qquad I_3{}^S\dot{\omega}_3{}^S = -P_S/\lambda_S \quad (64)$$

and recognizing the right-hand sides as the first approximation torques about the spin axes of bodies A and S. (This identification requires further use of the "averaging" procedure for asymmetric bodies.) Since it has been assumed that motor and bearing friction torques are arbitrarily small, only the damping mechanisms on bodies A and S can apply such torques. This correlation is demonstrated in more detail in the aforementioned reports.[10,11]

Once this relationship is accepted, it follows that

$$P_A < 0 \qquad P_S < 0 \quad (65)$$

Then substitution of Eq. (64) into Eq. (57) yields

$$\omega_0\dot{\omega}_0 = \left(\frac{P_A}{\lambda_A} + \frac{P_S}{\lambda_S}\right)\left(\frac{2h_0\lambda_2}{I_1{}^2\lambda_1 + I_2{}^2\lambda_2}\right) \quad (66)$$

As a necessary and sufficient condition for stability

$$\omega_0\dot{\omega}_0 < 0 \quad (67)$$

and since $\lambda_1\lambda_2 > 0$ is also necessary for stability [see Eq. (47)], and $h_0 > 0$ by convention, Eqs. (67) and (66) together provide

$$(P_A/\lambda_A) + (P_S/\lambda_S) < 0 \quad (68)$$

as a necessary and sufficient condition for stability of the dual spin motion, within the limitations of the present approximate analysis. Since P_A and P_S are negative, it is sufficient (but not necessary) for stability to have $\lambda_A > 0$ and $\lambda_S > 0$. Alternatively, one of these expressions may be positive and the other negative, as long as the dissipation rate in the body corresponding to positive λ is sufficiently high. This provides the quantitative basis for "trade-off" that is necessary for damper design.

Since λ_A and λ_S are the critical parameters for stability, and both are defined in Eq. (61) in terms of λ_0, this parameter may profitably be studied further.

From the definition (59), and the definitions of λ_1 and λ_2 in Eq. (26), λ_0 may be written

$$\begin{aligned}\lambda_0 &= h_0\left[\frac{h_0 - I_2\omega_A + h_0 - I_1\omega_A}{I_1h_0 - I_1I_2\omega_A + I_2h_0 - I_1I_2\omega_A}\right]\\ &= h_0\left[\frac{h_0 - \bar{I}\omega_A}{h_0\bar{I} - I_1I_2\omega_A}\right]\\ \lambda_0 &= \frac{h_0}{\bar{I}}\left[\frac{h_0 - \bar{I}\omega_A}{h_0 - (I_1I_2/\bar{I})\omega_A}\right]\end{aligned} \quad (69)$$

where $\bar{I} \equiv (I_1 + I_2)/2$

Several limiting cases can now be considered. If body A were symmetric (as is body S), so $I_1 = I_2 \equiv I$, and

$$(\lambda_0)_{\text{sym}} = h_0/I \quad (70)$$

then the stability parameters would become

$$\begin{aligned}(\lambda_A)_{\text{sym}} &= (h_0/I) - \omega_A\\ (\lambda_S)_{\text{sym}} &= (h_0/I) - \omega_S\end{aligned} \quad (71)$$

As a second special case, consider that of the partially "despun" system, for which $\omega_A = 0$. Then λ_0 is formed from the average transverse inertias, i.e.,

$$\lambda_0 = h_0/\bar{I} = I_3{}^S\omega_S/\bar{I} \tag{72}$$

and the stability parameters are

$$\lambda_A = I_3{}^S\omega_S/\bar{I} \tag{73}$$

and

$$\lambda_S = \lambda_0 - \omega_S = [I_3{}^S\omega_S - \bar{I}\omega_S]/\bar{I}$$

Clearly any energy dissipation in the despun portion of the system exerts a stabilizing influence, whereas energy losses in the spinning element are disadvantageous to stability if the rotor spin inertia is less than the average total transverse inertia. The result (68) can also be shown[10,11] to conform in another limiting case to the results of the Routh stability analysis in the preceding section.

Corroboration and Conclusion

It may be emphasized that the two preceding sections represent analytical approaches of distinctly different levels of rigor and generality. The arguments based on the energy-sink idealization should be accepted as heuristic and not considered to represent a formal proof of their conclusions. The Routh analysis does, however, establish rigorously the validity of these conclusions for a particular vehicle idealization, and adds credibility to the criteria obtained informally for more general systems. For further corroboration, simulations are useful.

One can never prove attitude stability by simulating a system in the laboratory or on a digital or analogue computer, since it is impossible to proceed by trial through all possible initial conditions in the neighborhood of the nominal motion. Yet until the abstractions and idealizations of the preceding analysis give way to the realities of the laboratory or the complexities of a digital computer numerical integration program, there remains a basis for skepticism regarding the applicability of analytical conclusions to physical systems. Consequently, the comprehensive study of dual spin systems reported here included both experimental and numerical aspects. These simulations included static and dynamic unbalance, and both accommodated internal closed-loop control systems and a motor driving relative rotation to accomplish pointing control, as well as attitude control jets to provide intermittent control over spin-axis attitude. All results conform to the analytical criteria established here. (See Ref. 11 for details.)

Although further analysis of dual spin systems may be expected (Floquet analysis and Liapunov analysis have yet to be reported), it may be safely concluded at this point that the validity and feasibility of dual spin stabilization systems have been firmly established.

References

[1] Pilkington, W. C., "Vehicle motions as inferred from radio signal strength records," Jet Propulsion Lab., Pasadena, Calif., External Publication 551 (September 5, 1958).

[2] Bracewell, R. N. and Garriott, O. K., "Rotation of artificial earth satellites," Nature **182,** 760–762 (September 20, 1958).

[3] Perkel, H., "Space vehicle attitude problems," Advanc. Astronaut. Sci. **4,** 173–192 (1958).

[4] Thomson, W. T. and Reiter, G. S., "Attitude drift of space vehicles," J. Astronaut. Sci. **7,** 29–34 (1960).

[5] Likins, P. W., "Modal method for analysis of free rotations of spacecraft," AIAA J. **5,** 1304–1308 (1966).

[6] Likins, P. W., "Effects of energy dissipation on the free body motions of spacecraft," Jet Propulsion Lab., Pasadena, Calif., Tech. Rept. 32-860 (July 1, 1966).

[7] Pringle, R., Jr., "On the stability of a body with connected moving parts," AIAA J. **4,** 1395–1404 (1966).

[8] Landon, V. D. and Stewart, B., "Nutational stability of an axisymmetric body containing a rotor," J. Spacecraft Rockets **1,** 682–684 (1964).

[9] Iorillo, A. J., "Nutation damping dynamics of axisymmetric rotor stabilized satellites," American Society of Mechanical Engineers Winter Meeting, Chicago, Ill. (November 1965).

[10] Likins, P. W., "Attitude stability of dual spin systems," Hughes Aircraft Co., El Segundo, Calif., SSD 60377R (September 1966).

[11] Velman, J. R., "Attitude dynamics of dual spin satellites," Hughes Aircraft Co., El Segundo, Calif., SSD 60419R (September 1966).

[12] Routh, E. J., *Advanced Dynamics of a System of Rigid Bodies* (The Macmillan Co., London, 1905), 6th ed.; reprinted by Dover Publications. Chap. VI, pp. 226, 227.

[13] Cesari, L., *Asymptotic Behavior and Stability Problems in Ordinary Differential Equations* (Academic Press, New York, 1963), 2nd ed., Chap. II, pp. 55.